Two-dimensional convection from heated wires at low Reynolds numbers

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Measurements of heat transfer from circular wires placed normal to a horizontal airstream have been made in the Reynolds number range 0·01 to 140. The Nusselt number can be related to the Reynolds number and temperature loading by an expression of the form

$$N\left(\frac{T_m}{T_\infty}\right)^{-0.17} = A + BR^n,$$

where the values of n, A and B (see table 3) depend on whether the Reynolds number is above or below the value for which a vortex street exists in the wake of the wire. This value of the Reynolds number ($R \doteqdot 44$) is independent of the intensity and scale of the stream turbulence. The theoretical heat transfer relation based on the Oseen approximation is approached asymptotically as $R \to 0$, provided free convection is negligible.

Free convection effects diminish rapidly with increasing Reynolds number so that the orientation of the wire with respect to the vertical has a negligible influence on heat transfer except at very low velocities. For horizontal wires at very low Reynolds numbers, free convection is significant, when, roughly speaking, the Reynolds number is less than the cube root of the Grashof number.

1. Introduction

Heat transfer from circular cylinders by forced convection has been widely studied because of its engineering importance. This paper is concerned only with the range of variables encompassed in instruments employing heated cylindrical wires as sensing elements (Corrsin 1949). The most important application of hot-wire instruments is probably in fluid velocity and turbulence measurements. Unfortunately, the many advantages of the hot-wire anemometer in such work are somewhat offset by large uncertainties in the absolute magnitude of measured quantities. This deficiency is in part attributable to the use of inaccurate heat-transfer relations. The experimental investigation described below has yielded a new general relation governing heat-transfer by two-dimensional forced convection in air which should assist in improving the accuracy of hot-wire measurements.

Dimensional analysis of the equations for convection of heat by an incompressible fluid (Goldstein 1938) shows that the dimensionless heat-transfer coefficient, or Nusselt number N, is a function of the flow parameters and fluid properties as follows:

$$N = f(R, G, \sigma), \tag{1}$$

where R is the Reynolds number, G the Grashof number, and σ is the Prandtl number. This analysis assumes that the fluid properties are independent of temperature and that it is a continuous medium. The former assumption is rarely valid, and it is customary to take account of this by introducing a parameter representing the temperature loading of the heated body, e.g. T_W/T_∞ or T_m/T_∞, where T denotes temperature and the suffixes W, ∞, m respectively signify conditions at the body surface, in the free stream, and the arithmetic mean of these. In the experiments reported here, some of the heated wires were sufficiently small in diameter for the effects of the molecular nature of air to be experienced. Molecular effects take the form of changed boundary conditions as compared with continuum flow—there is a jump in temperature between the surface of the wire and the gas adjacent to it, and the fluid slips or moves over the surface with a finite velocity. In rarefied gas dynamics the ratio of mean free path to cylinder-diameter, i.e. the Knudsen number K, is recognized as the parameter characterizing molecular effects. Thus, for the complete specification of convective heat transfer from fine hot wires, a relation of the following type is required:

$$N = f(R, G, \sigma, K, T_m/T_\infty). \tag{2}$$

There is still insufficient knowledge of the subject to permit the formulation of such a relation, and in any case it would be too unwieldy for practical purposes. In practice, free and forced convection are treated separately. As a result of recent investigations (e.g. Collis & Williams 1954), free convection from wires in air can be specified in the form

$$N = f(G, T_m/T_\infty) \tag{3}$$

for Grashof numbers as small as 10^{-10}. It has been further reported (Beckers *et al.* 1956) that free convection at Grashof numbers less than about unity is independent of Prandtl number, so that equation (3) is valid for other fluids in this case. Forced convection from hot wires in air has long been specified by simple relations of the following type, or by its equivalent in dimensional variables:

$$N = f(R, T_m/T_\infty). \tag{4}$$

The results of the present measurements are also given in this form. The variation of Prandtl number of air with temperature is small and can be combined with other temperature loading effects. Explicit introduction of the parameter K has been dispensed with, because molecular effects are small enough to be treated as a correction to the continuum heat transfer coefficient. This correction, described in Appendix A, is made on the assumption that temperature jump is solely responsible for the observed reduction in the Nusselt number. Rough estimates suggest that the effect of velocity slip would lie between $\frac{1}{3}$ and $\frac{1}{5}$ of the jump effect in the present experiments.

At sufficiently low Reynolds numbers, forced and free convection interact in a manner which has not yet been fully investigated. Ower & Johansen (1931) and Cooper & Linton (see Ower 1949) have described this interaction, essentially in a qualitative manner. In the particular case of horizontal wires in a horizontal air stream, sufficient data was obtained in the present series of experiments to permit the formulation of a criterion for the onset of significant free convection as the Reynolds number is decreased from values where forced convection predominates. Further, it has been clearly shown that at higher Reynolds numbers beyond this small region of interaction, the effect of free convection is quite negligible. This observation shows that the method of hot-wire anemometer calibration which involves the measurement of heat transfer at zero Reynolds number is erroneously based.

Since the primary aim of this work was to establish precise heat transfer laws for forced convection at low Reynolds numbers the existing information on this subject will now be examined in more detail.

2. Review of data on forced convection at low Reynolds numbers

The results of the more important forced convection investigations have been collected and correlated by McAdams (1954). The scatter of results is rather large, but a correlation between N and R, satisfactory for many engineering purposes, has been achieved for temperature loadings up to 1000 °C by evaluating the thermal conductivity k and the dynamic viscosity μ of the gas at a temperature halfway between the cylinder temperature and the ambient or free-stream temperature, i.e. $\frac{1}{2}(T_W + T_\infty)$, and the density ρ at free stream temperature. In the Reynolds number range 0·1 to 250,000, McAdams gives co-ordinates of a recommended mean curve, whilst for the smaller range 0·1 to 1000, he gives the equation

$$N = 0{\cdot}32 + 0{\cdot}43R^{0{\cdot}52}. \tag{5}$$

Mean equations such as this smooth out the finer details, and therefore are too crude to form the basis for accurate measuring devices, particularly those which, like the turbulence hot wire, depend on the values of $\partial N/\partial R$ at the operating point. It is probable that a single careful investigation, covering perhaps a more restricted range of the variables, would yield more satisfactory heat-transfer relations.

Of the various sources quoted by McAdams, only two contribute extensive data at Reynolds numbers in the range of operation of hot-wire instruments (viz. $R < 100$), namely King (1914) and Hilpert (1933). Hot-wire anemometry is almost invariably based on an empirical relation obtained by King. If a wire of diameter d, placed normal to an airstream of velocity V, and temperature T_∞, is heated to a temperature T_W, then the heat transfer coefficient H W cm^{-1} deg^{-1} as given by King's law is

$$H = A[1+\gamma(T_W - T_\infty)] + B[1+\delta(T_W - T_\infty)]\sqrt{(Vd)}, \tag{6}$$

where $A = 2{\cdot}50\times 10^{-4}(1+35d)$ with d in cm, $B = 1{\cdot}012\times 10^{-2}\sqrt{d}$, and the temperature coefficients when $T_\infty = 17$ °C are $\gamma = 0{\cdot}00114$, $\delta = 0{\cdot}00008$.

King's experiments were performed with a whirling arm and consequently were subject to interference from draughts, both natural and induced. However, more recent investigations in wind tunnels or air jets of low turbulence level have been found to confirm King's law, but these investigations have generally covered a very limited range of the variables, and it is doubtful whether the results have been examined sufficiently critically. It will be shown later, in fact, that King's results involved some fairly large systematic errors, and that because of this the results can be interpreted as being consistent with the proposed new heat transfer relation which differs somewhat from equation (6).

The work of Hilpert is exceptional in that it extends over a very wide Reynolds number range and exhibits a high degree of consistency throughout. Hilpert expressed his results in the form of a power law

$$N = C\left[R\left(\frac{T_W}{T_\infty}\right)^{\frac{1}{4}}\right]^n, \tag{7}$$

where C and n are constant in specified ranges of R. The method of correlation in this equation differs from that of McAdams in that the gas density as well as the conductivity and viscosity are evaluated at the mean temperature,† $\frac{1}{2}(T_W + T_\infty)$. Table 1 contains values of C and n for the ranges of interest here.

R	C	n
1–4	0·891	0·330
4–40	0·821	0·385
40–4000	0·615	0·466

TABLE 1

There are two reasons which possibly account for the fact that the results have not been applied extensively in instrument work. First, the temperature factor was based on a series of measurements in which the Reynolds number varied only through a small range. It was therefore not shown that this factor retains the same form in other ranges of Reynolds number, although some support was derived by correlating some of King's data at lower Reynolds numbers. Secondly, the series of simple power laws is probably only an approximate representation of the data, which could lead to appreciable errors in deriving the local slope of the heat-transfer curve. Although the experimental results were published in tabular form, no alternative analysis seems to have been attempted, and the discontinuities in the slope of the heat-transfer curve which Hilpert noted at $R = 4$ and $R = 40$, have been generally ignored in instrument work. The second of these irregularities obviously could arise from the sudden change in nature of the flow, which occurs when vortices begin to detach from the rear of the cylinder. The former, which was less well defined, might be due to the formation of standing vortices—a more gradual process.

A theoretical solution to the forced convection problem for Reynolds numbers in the range of 0·1 to 100 is extremely difficult, since neither the slow viscous flow approach on the one hand, nor the use of boundary layer approximations on the other, is allowable for most of the range. The one well-known solution for forced

† Allowance for this difference has not been made by McAdams (1954) in incorporating Hilpert's results in McAdams' figure 10–7. The required adjustment amounts to an increase of $13\frac{1}{2}$ % in Reynolds number. A rather smaller adjustment to McAdams' recommended curve seems indicated.

convection from wires, that due to King, avoids the difficulty by assuming potential flow. King's solution is somewhat tedious to evaluate, but can be approximated closely by two equations of simpler form. Written in non-dimensional terms these are†

$$N = \frac{2}{\log(2e^{1-\gamma_E}/R\sigma)} \qquad (R\sigma < 0{\cdot}08), \tag{8}$$

where $\gamma_E = 0{\cdot}577\ldots$ is Euler's constant, and

$$N = \frac{1}{\pi} + \sqrt{\left(\frac{2}{\pi}R\sigma\right)} \qquad (R\sigma > 0{\cdot}08). \tag{9}$$

The second equation is identical in form with King's experimental law, except that it does not include the effect of temperature on the physical properties of the gas. Comparison with experimental data shows that it overestimates the heat transfer by up to 40 %. Apart from this, the basic assumptions of the theory have been held to be unsatisfactory (Goldstein 1938). The theory thus is of no help in assessing the relative merits of experimental results.

A useful piece of theory (although of limited scope) is that based on the Oseen approximation. This is of course strictly valid only in the limit as $R \to 0$. Cole & Roshko (1954) find that this approximation yields the solution

$$\frac{2}{N} = \log\frac{8}{R\sigma} - \gamma_E. \tag{10}$$

Experimental work by the same authors failed to substantiate equation (10) due, it was thought, to disturbance of the two-dimensional convection by three-dimensional effects.

The investigation which is now described was provoked by observations that hot-wire anemometer calibrations made under near-ideal conditions tended to deviate systematically, and always in the same manner, from King's law (equation (6)). The nature of these deviations was such as to indicate a somewhat different dependence of heat-transfer on velocity from that of equation (6). A modification of King's law is proposed in this paper following careful measurements involving a wide range of air velocities, wire diameters and temperature loading. The existence of at least one discontinuity in the heat transfer relation as previously observed by Hilpert has been verified. The dependence on temperature loading was found to agree in magnitude with Hilpert's finding rather than with that of King.

3. Description of measurements

3.1. *Equipment*

Heat transfer measurements in the speed range 6–140 ft./sec were made with electrically heated wires in a small closed-return wind-tunnel (test-section area $10\frac{1}{2} \times 13\frac{1}{2}$ in.) of conventional design. An excellent distribution of velocity and a low turbulence level were obtained by using a single wire gauze screen in the

† The specific heat at constant volume has been replaced by that at constant pressure, which seems to be more appropriate in this problem.

settling chamber followed by a 4:1 contraction of area. The longitudinal component of turbulence was found to be 0·08 % at 50 ft./sec. Air speed was determined from the pressure drop across the contraction, which was calibrated by means of a Pitot-static tube placed at the position later occupied by the heated wires. The temperature of the air in the tunnel was slightly sensitive to changes in atmospheric conditions, and also changed several degrees during runs at the higher speeds due to viscous dissipation. As heat-transfer measurements were made for temperature loadings as small as 30 °C, a check on changes in ambient temperature was thus essential. This was made by means of a rapid response, platinum thermometer placed near to, but clear of, the wake of the heated wire. From time to time an ambient temperature reading was obtained from the test wire itself. Comparison of such readings with those of the monitoring thermometer served as a check on changes in the test wire arising from strain or other causes. Ambient temperature varied between 10 and 25 °C during the experiments.

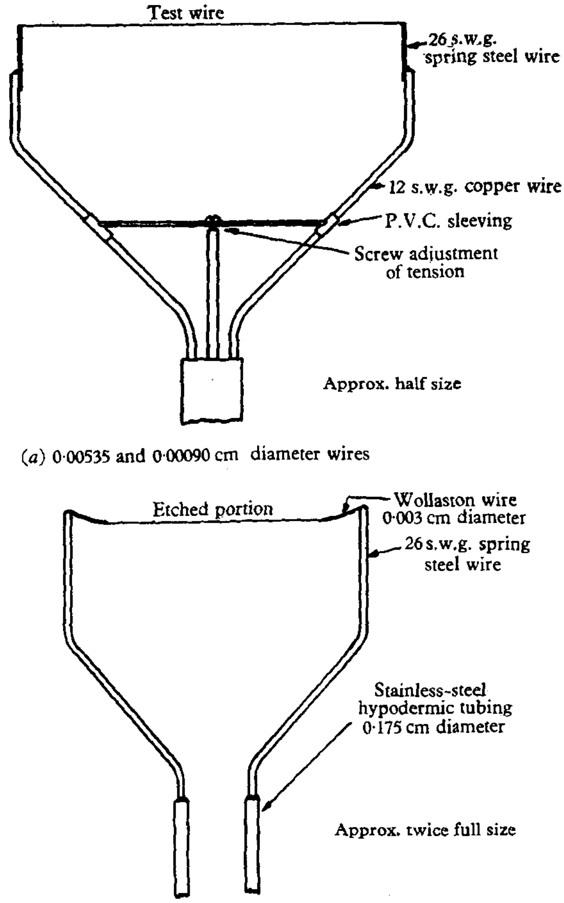
For measurements at lower air speeds, namely, from 0·08 to 1·42 ft./sec, a cylindrical duct fitted at the intake end with a bell mouth, honeycomb and gauze was used. Air speed was measured by means of a Simmons' shielded anemometer (Simmons 1949), which was originally calibrated on a whirling arm at the National Physical Laboratory.† Flow conditions in this duct were much less steady than in the wind-tunnel, and velocity measurements were less accurate. Thus, in the experimental results presented later, there is a greater scatter amongst data taken at speeds less than 6 ft./sec.

3.2. *Wire assemblies*

In some earlier work in this field the effect of finite wire length has been largely eliminated by attaching potential taps to the heated wire, so that end losses are not included in the measured heat transfer. This device is not practicable for the finer wire used in the present work. The procedure adopted was to use very long wires and correct the results for the small residual transmission of heat to the supports by the method given by Simmons & Beavan (1934). The maximum correction necessary was 3·6 %.

The heavier gauges of wire were mounted by soldering them between slender steel prongs as in figure 1 (*a*). Tension was applied subsequently by a screw adjustment. The finest gauge wires which were prepared from Wollaston wire could not be mounted under tension without risk of breakage by vibration. An arrangement in which tension is maintained by air-drag was therefore used. An assembly of this type is shown in figure 1 (*b*). The Wollaston wire is soldered in a slack loop trailing downstream of the prongs. The silver layer is then etched from the centre of the loop and the platinum is straightened by manipulation of the remaining silver supports. At the maximum speed at which the finest wire was used, namely, 100 ft./sec, the platinum was deformed into a catenary with the tangent at either end inclined at about 5° to the normal to the air stream. This deformation would have a negligible effect on the rate of heat transfer.

† In using the Simmons' anemometer as a transfer device, no assumption about the heat transfer law applicable to this instrument is involved.



(*a*) 0·00535 and 0·00090 cm diameter wires

(*b*) 0·000295 cm diameter wire

FIGURE 1. Wire support geometry.

3.3. *Properties of the heated wires*

Wires of nominally pure platinum were employed for all of the measurements. The dimensions of each wire and the temperature coefficient of electrical resistance of the material are given in table 2. An optical interference method was used to determine the diameters of the finer specimens of wire whilst a similar technique combined with the use of slip gauges was used for the thickest wire. Accuracy of the two methods was about 2 and 1 %, respectively. A travelling microscope was used for the length measurements. The temperature coefficients of resistance were measured over the interval 0–100 °C, using melting ice and steam baths. The variation of this coefficient is indicative of impurities in the platinum.

The length of each specimen was made as great as practicable to minimize both the metallic conduction heat transfer and other three-dimensional effects. Free convection measurements (Collis & Williams 1954) have shown that all end effects (including metallic conduction) are small for $l/d \geqslant 20{,}000$, even at very small values of the Nusselt number. As the Nusselt number increases the hot air film diminishes in thickness, and rough calculations based on the Langmuir concept of a concentric cylindrical film suggested that $l/d > 1000$ would be adequate for these experiments. However, the concentric film concept is not valid for forced convection, so that in the absence of a more reliable guide a large margin of safety in the aspect ratio was considered desirable. The most critical condition occurred with the finest gauge wire at the lowest velocities. The specimen with $l/d = 5370$ was therefore used for all measurements at speeds less than 6 ft./sec. At higher speeds most of the measurements were made with a shorter, and therefore stronger specimen in order to minimize errors arising from strain and permanent deformation of the wire.

Diameter d (cm)	Length l (cm)	Aspect ratio l/d	Temp. coeff. of resistance
0·000295	0·872	2950	0·00370
0·000295	1·585	5370	0·00370
0·00090	7·80	8660	0·00380
0·00535	11·10	2070	0·00342

TABLE 2

3.4. *Measurement of heat-transfer coefficients*

The wires under test were heated electrically and the power dissipated in the wire was computed from measurements of the heating current and wire resistance. The heat lost by convection is equated to the electrical power dissipation, less losses to the supporting wires. The temperature loading was determined by using the test wire as an electrical resistance thermometer. Temperatures on the International Temperature Scale were obtained by first calculating temperatures on the platinum scale using the measured temperature coefficients and then applying a correction by the Callendar method (Kaye & Laby 1948), assuming the difference coefficient $\delta = 1{\cdot}5$. Resistance of the wire at ambient temperature was determined by extrapolating to zero power dissipation. Ambient temperature was measured on a mercury thermometer prior to a run and corrected for rapid changes by reference to the platinum thermometer referred to in § 3.1.

Radiation losses were computed and found to be negligible throughout.

3.5. *Eddy shedding observations*

A hot wire anemometer was mounted in the wake of an unheated wire, of 0·00535 cm diameter, and a turbulence amplifier with a useful frequency range up to about 70 kilocycles per second was used to detect eddies shed from the upstream wire. Turbulence level and scale were varied by inserting square-mesh wire grids into the working section upstream of the wires. Determinations of the Reynolds number at which eddy shedding commenced was made for a range of turbulence conditions.

4. Results

The results discussed in §§4.1 to 4.4 refer to horizontal wires in a horizontal airstream normal to the wire. The effect of orientation is considered in §4.5.

4.1. *Method of correlation*

When formulating empirical relationships it is customary to attempt to remove explicit dependence of the Nusselt number on temperature loading by evaluating the physical properties of the fluid at some suitable temperature. Choosing this temperature on the grounds of physical significance does not always lead to a unique simple relationship between the variables. Here, the system used by Hilpert has been followed (except where otherwise stated) as it leads to a compact expression of the results in the Reynolds number range concerned. Thus, the fluid properties—thermal conductivity, density and viscosity—are evaluated at a temperature which is the arithmetic mean of the free stream (or ambient) and cylinder temperatures. This system, it will be recalled, differs from that used by McAdams, who evaluated density at the free stream temperature.

Values of the thermal conductivity of air were computed from a formula given by Kannuluik & Carman (1951) which, for thermal conductivity in electrical units, is

$$k_t = 2{\cdot}41 \times 10^{-4}(1 + 0{\cdot}00317t - 0{\cdot}0000021t^2), \tag{11}$$

where k_t W cm^{-1} deg.$^{-1}$ is the thermal conductivity at temperature t °C. The ratio of viscosity to density of air, i.e. the kinematic viscosity μ/ρ, was obtained from tables in Goldstein (1938).

4.2. *Correlation of forced convection data*

The range of Knudsen number K, involved in these measurements is too small to attempt an accurate evaluation of the effect of temperature jump on heat transfer. However, in order to reduce all of the data to comparable conditions a correction was applied to reduce the measured heat-transfer coefficients to continuum values. Figure 2 has been drawn to exhibit the magnitude of the correction necessitated by temperature jump. A low-temperature loading was chosen for this illustration to avoid complications arising from temperature dependence of the fluid properties. The derivation of a correction for temperature jump based on kinetic theory is given in Appendix A. It is assumed that the measured rate of heat transfer is that which would occur in a continuum under the temperature difference which exists in the gas outside the region of discontinuity. The continuum Nusselt number N_C is then related approximately to the apparent or measured value N_M in the following way for low temperature loadings:

$$\frac{1}{N_M} - \frac{1}{N_C} = 2K. \tag{12}$$

For high-temperature loadings, allowance must be made for the temperature dependence of mean free path and thermal conductivity as shown in Appendix A.

Applying the correction enables all data pertaining solely to forced convection to be consolidated into a single curve for each temperature loading. This is demonstrated on a logarithmic scale for a low- and a high-temperature loading by means of figure 3. The series of points which diverge sharply from the upper forced convection curve at low Reynolds numbers are associated with heat transfer due to mixed free and forced convection. The conditions under which free convection becomes important are discussed in a later paragraph. The

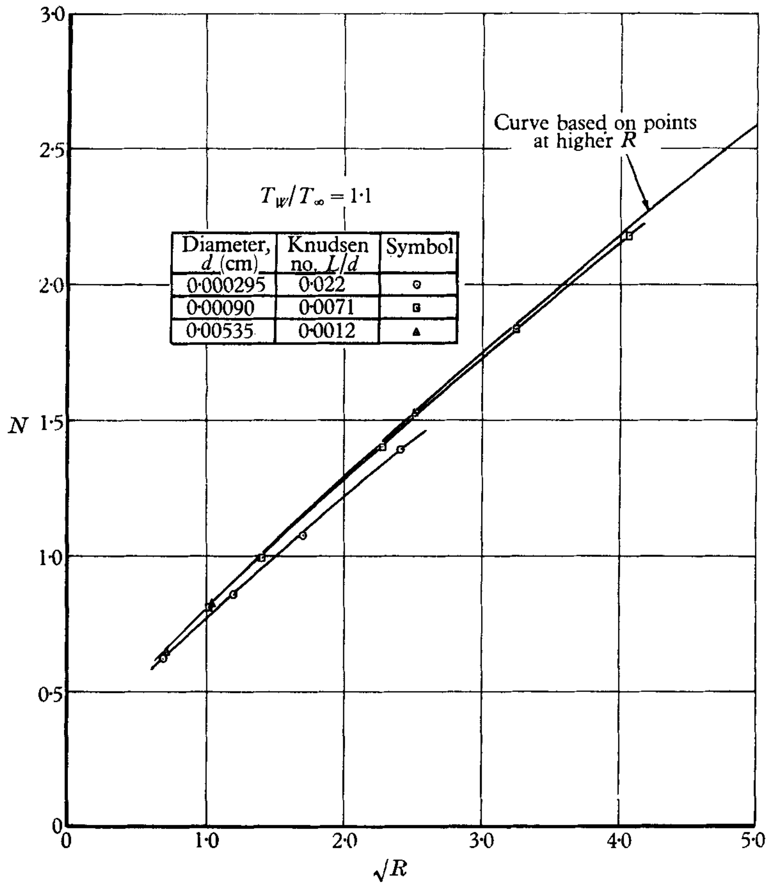


FIGURE 2. Effect of temperature jump on heat transfer.

evaluation of fluid properties at the arithmetic mean temperature T_m clearly does not entirely eliminate the temperature loading T_W/T_∞ or T_m/T_∞ as a necessary parameter. Nevertheless, the residual effect of temperature loading is small and is essentially independent of Reynolds number. The effect amounts to roughly 7 % increase in Nusselt number for an increase in temperature difference from 30 to 320 °C. Thus, for the range of variables covered in this work, the dimensionless heat-transfer coefficient, temperature loading and mass flow can be correlated by an equation of the form

$$N . g\left(\frac{T_m}{T_\infty}\right) = f(R), \tag{13}$$

where g and f are functions to be determined. A satisfactory form for the temperature function is

$$g\left(\frac{T_m}{T_\infty}\right) = \left(\frac{T_m}{T_\infty}\right)^{-0\cdot17} \tag{14}$$

It should be noted that this function is somewhat dependent on the values chosen for the thermal conductivity, and various sources differ somewhat in their values at elevated temperatures. In figure 4 the effectiveness of relations (13) and (14) in condensing the whole body of forced convection data into a single curve is demonstrated.

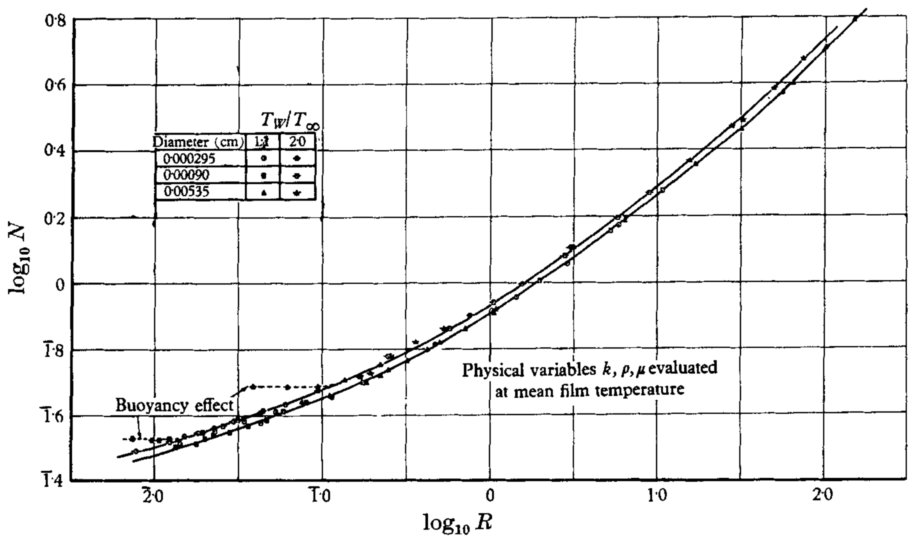


FIGURE 3. Variation of continuum Nusselt number with Reynolds number showing residual effect of temperature loading.

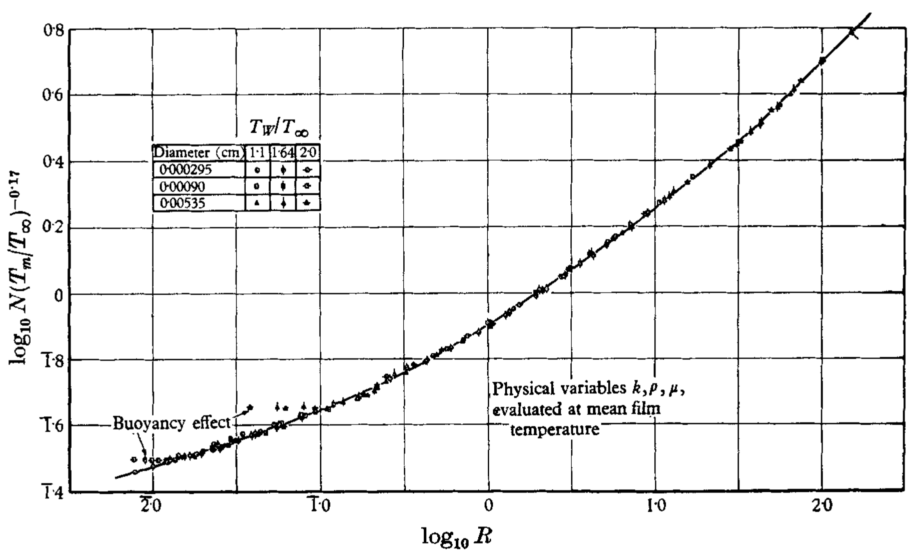


FIGURE 4. Nusselt number uniquely correlated with Reynolds number and temperature loading.

The next step is to find a convenient analytical approximation to the flow function $f(R)$. Since it is usually accepted that the data conform to King's equation (with modified constants), it will be instructive to make the comparison, remembering that the investigation was undertaken because of suspected departures from King's law. Figure 5 shows clearly that the supposed linear

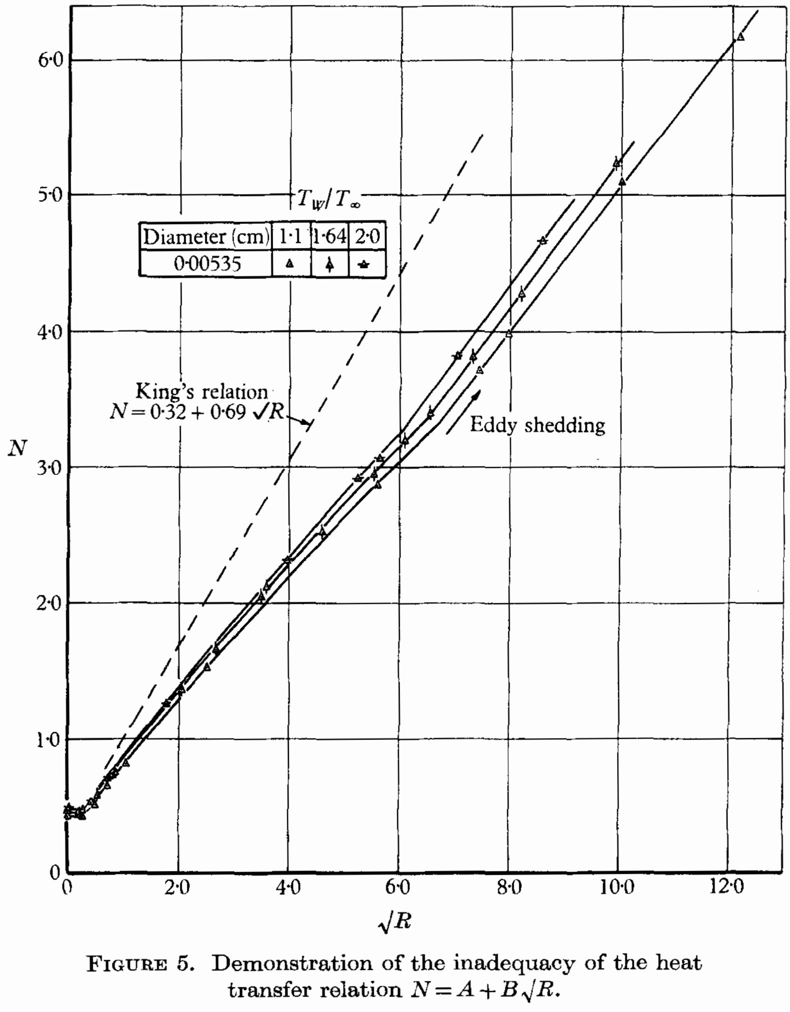


FIGURE 5. Demonstration of the inadequacy of the heat transfer relation $N = A + B\sqrt{R}$.

relation between N and $\sqrt{R}$ is no more than a rough approximation at Reynolds numbers less than about 44. Thus, the constants of King's law as determined by experiment must be somewhat arbitrary, because of the latitude available in approximating the curves of figure 5 by straight lines. In particular the slope of such lines is very variable, which explains a great deal of the inconsistency common in turbulence measurements by hot wire. At Reynolds numbers greater than 44 or thereabouts, eddies are shed from the rear of the cylinder. The change in the velocity distribution, which occurs abruptly, gives rise to the marked change of slope of the heat-loss curves which can be seen in figure 5, at about

$\sqrt{R} = 6{\cdot}6$. The function $f(R)$ thus has some sort of discontinuity at this critical Reynolds number. By trial and error it was found that the experimental data were in good agreement with the equation

$$N\left(\frac{T_m}{T_\infty}\right)^{-0\cdot17} = A + BR^n \tag{15}$$

if the constants A, B, n have the values given in table 3 in the Reynolds number ranges prescribed there. Most of the data shown in figure 4 are replotted in figure 6, showing how the 0·45 power law is satisfied in the range specified in table 3. The change following the onset of eddy shedding is clearly defined. At

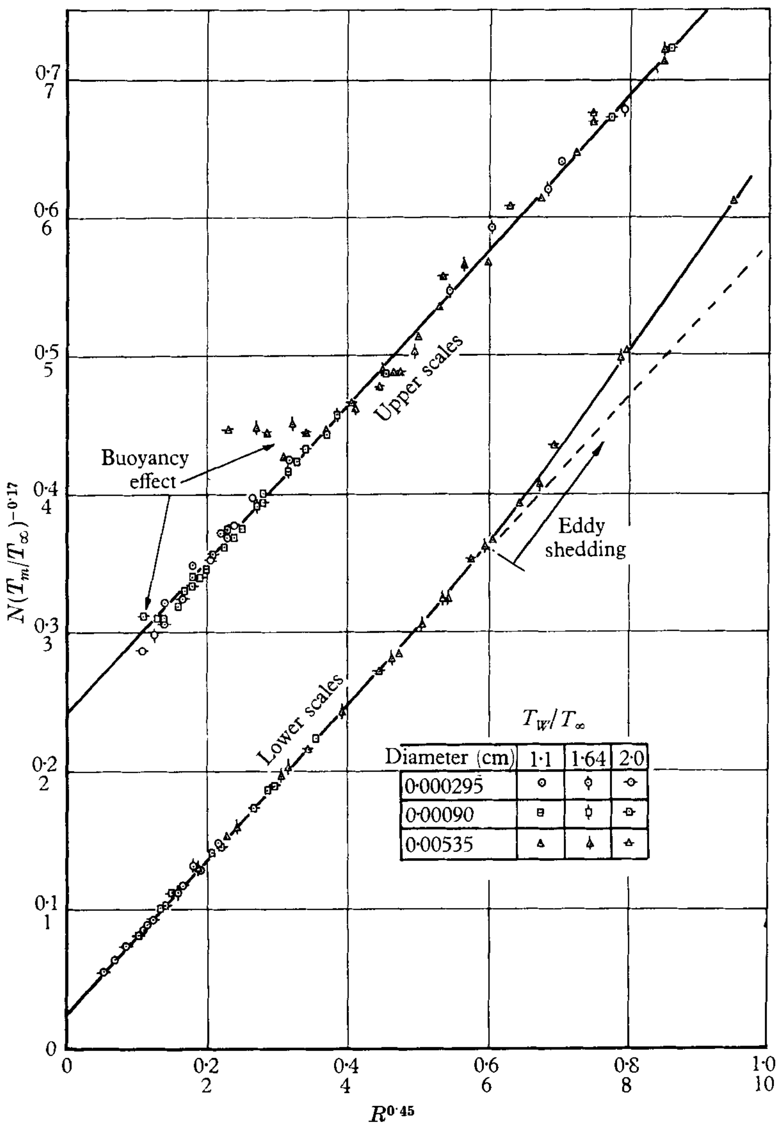


FIGURE 6. Heat transfer data correlated by 0·45 power law in the range $0{\cdot}02 < R < 44$.

$R = 0{\cdot}02$, equation (15), with appropriate constants, overestimates the Nusselt number by 2 or 3 % and the error increases rapidly if the same relation is used at smaller Reynolds numbers.

	$0{\cdot}02 < R < 44$	$44 < R < 140$
n	0·45	0·51
A	0·24	0
B	0·56	0·48

TABLE 3

4.3. *Forced convection at very low Reynolds numbers*

The theoretical solution for two-dimensional forced convection from cylinders obtained by Cole & Roshko (1954) using the Oseen method, is believed to be correct in the limit as $R \to 0$, and should not be greatly in error for $R \ll 1$. A comparison of the experimental results at low Reynolds numbers, with the Oseen solution, thus provides a check on the general soundness of the measurements and in particular of their approach to two-dimensionality. This comparison is made in figure 7 for large and small temperature loadings. The straight (broken) line represents the Oseen solution (equation (10)). Some data depart noticeably

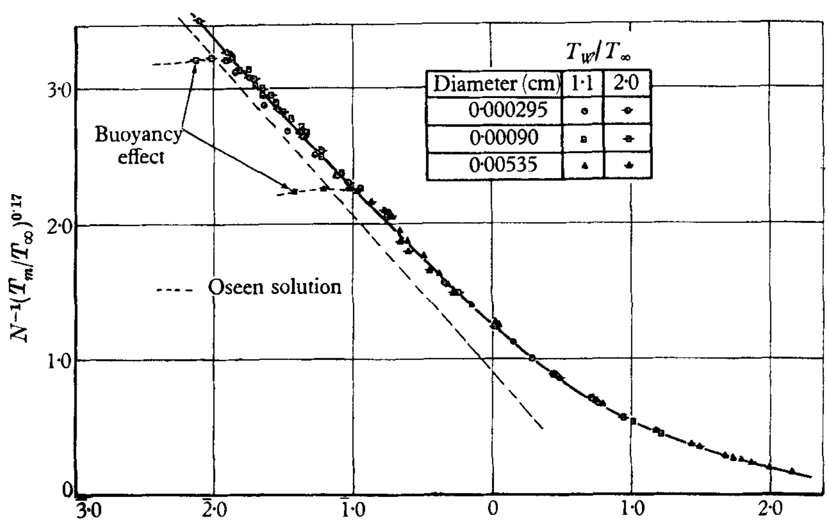


FIGURE 7. Comparison with Oseen solution.

from the forced convection curve because of buoyancy effects, but the smooth curve through the bulk of the points asymptotes to the Oseen solution as $R \to 0$. This curve deviates from the theoretical one by about 5 % at $R = 0{\cdot}01$, and diverges steadily as the Reynolds number increases, as would be expected. The discrepancy is thus too great for the Oseen solution to have much value as a working rule for forced convection at low Reynolds numbers. However, a relation of similar form can be used to describe the experimental results for $R < 0{\cdot}5$ with an error less than the experimental scatter:

$$N\left(\frac{T_m}{T_\infty}\right)^{-0{\cdot}17} = \frac{1}{1{\cdot}18 - 1{\cdot}10\log_{10} R}. \tag{16}$$

This equation agrees with equation (15) to within 2 % in the range $0{\cdot}02 < R < 0{\cdot}5$, and is in better agreement with experiment at $R \leqslant 0{\cdot}02$ than is equation (15). Equation (16) will almost certainly be a good approximation for two-dimensional heat transfer at any lower Reynolds numbers which are attainable under continuum conditions (or nearly such) and in the absence of significant buoyancy effects.

4.4. *Mixed free and forced convection*

As we have noted, certain series of points diverge quite sharply from the forced convection curve near the lower end of the Reynolds number range. This divergence occurs because the components of velocity, induced in the heated

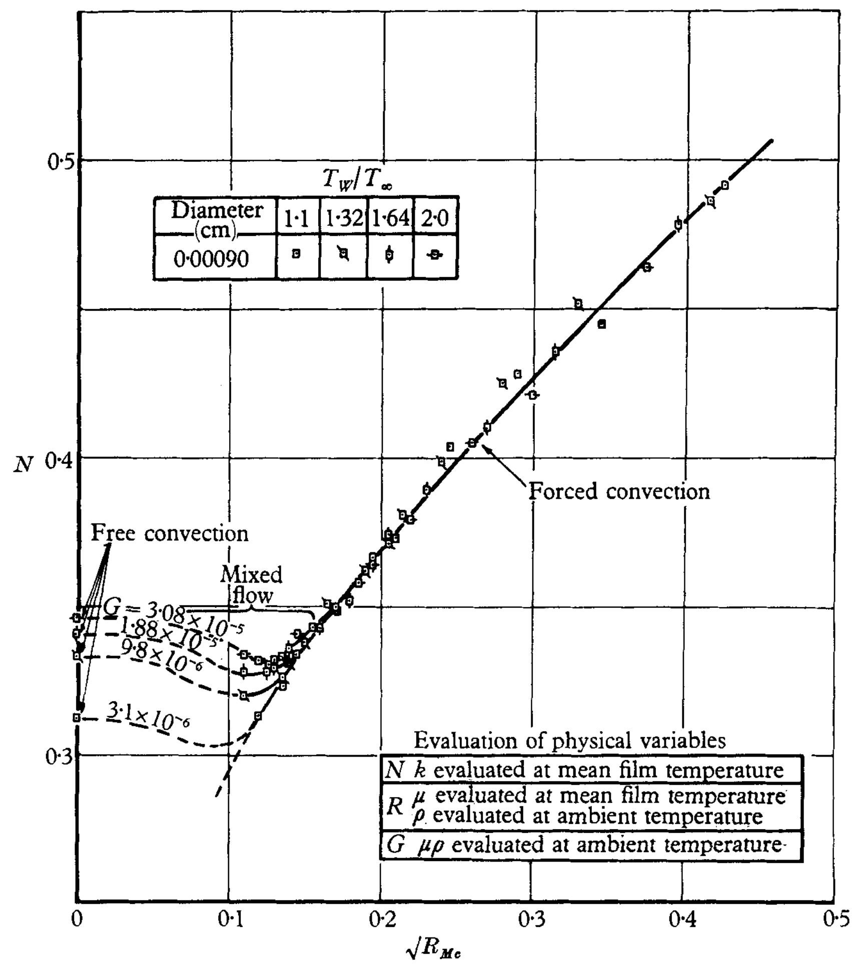


FIGURE 8. Interaction of free and forced convection.

fluid adjacent to the cylinder by buoyancy forces, become comparable in magnitude with components of the forced draught. Figures 3, 4, 6 and 7 show that the points of divergence depend markedly on cylinder diameter. By plotting the very low Reynolds number data on an enlarged scale as in figure 8, a temperature dependence is also observed. In this figure the method of correlation has been modified to the extent that the gas density embodied in the Reynolds number is evaluated at ambient temperature after the practice of McAdams (1954). This allows the temperature function of equation (15) to be discarded for the particular

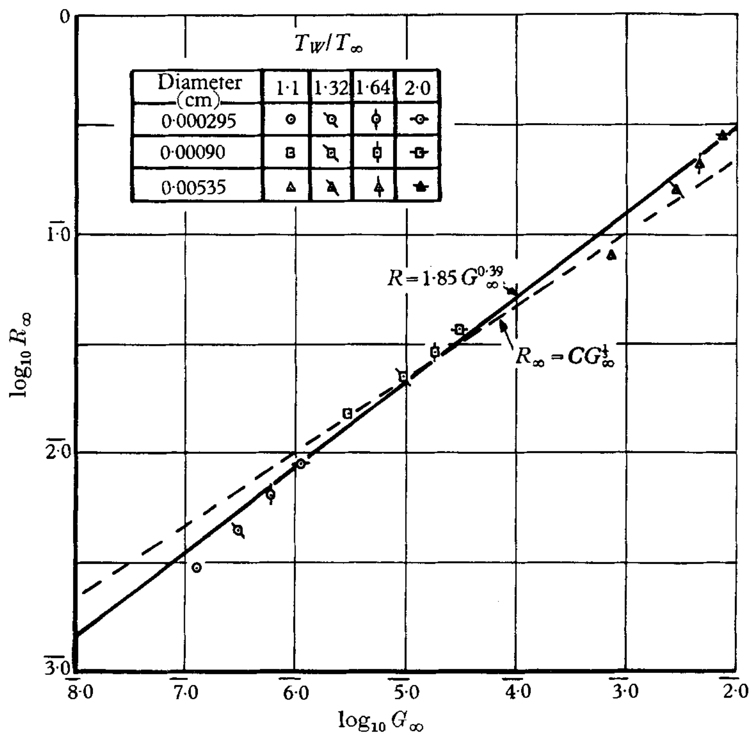


FIGURE 9. Criterion for onset of buoyancy effect.

small range of Reynolds numbers involved—a fact which will prove convenient presently (Appendix B). The data pertain to one wire diameter only, but the buoyancy effect sets in at different Reynolds numbers for each temperature. It is also clearly shown that, as the Reynolds number is further reduced, the heat-transfer rate passes through a shallow minimum before reaching the free convection value at zero Reynolds number. This curious phenomenon was apparently first observed by Ower & Johansen (1931) and later confirmed by others. However, except for special cases, the conditions governing the occurrence of mixed free and forced convection do not appear to have been quantitatively determined. Some information on this question can now be derived.

Since free convection and forced convection in a particular fluid (σ = const.) depend primarily on the Grashof and Reynolds numbers, respectively, a criterion for mixed flow involving only these two parameters should exist. This is consistent qualitatively with the observed dependence on diameter and temperature. As a characteristic point of the mixed flow régime, we have chosen the condition for which the Nusselt number of mixed convection equals that for pure free convection at the same Grashof number. This point has the practical significance that it defines the lowest Reynolds number at which the hot wire in question can be used as an anemometer without ambiguity. Furthermore, the trend of the data with increasing Reynolds numbers indicates that the effect of buoyancy on the forced convection rapidly becomes negligible. In figure 9, the values of Reynolds and Grashof numbers, associated with these points of equal Nusselt number, have been plotted. Points pertaining to the finest wire could only be obtained by extrapolation. Giving less weight to these points, the conclusion was reached that buoyancy effects are small provided†

$$R_\infty > G_\infty^{\frac{1}{3}}. \tag{17}$$

The suffixes denote evaluation of the fluid properties at ambient temperature, but the conclusion, being very approximate, is not sensitive to the temperature dependence of these properties.

Another criterion based on a greater volume of data is obtained in Appendix B, and is probably more accurate for a limited range of Reynolds number. It is derived by assuming that there is no deviation from the pure forced convection relation (equation (16)) due to buoyancy effect until the Nusselt number reaches that due to free convection alone. The details of this analysis are given in Appendix B. The criterion derived for Reynolds numbers less than about 0·1 is that

$$R_\infty = 1{\cdot}85\, G_\infty^{0{\cdot}39} \left(\frac{T_m}{T_\infty}\right)^{0{\cdot}76}. \tag{18}$$

The straight line corresponding to zero temperature loading ($T_m/T_\infty = 1$) is shown in figure 9. The overall trend of the experimental points is followed, but in general the Reynolds number predicted is too high for a given Grashof number because of the assumption of no-deviation from the forced convection relation (16). The groups of points belonging to two of the three wires show additional dependence on temperature in good agreement with equation (18). Apart from the numerical factor it seems that equation (18) provides a fair criterion for the onset of buoyancy effects. In particular, it follows that

$$V_{\min} \doteqdot \text{const.} \times d^{\frac{1}{5}} (T_W - T_\infty)^{\frac{2}{5}} \left(\frac{T_m}{T_\infty}\right)^{\frac{3}{4}}, \tag{19}$$

where $V_{\min}$ is the lowest velocity which can be measured without ambiguity by a hot wire of large aspect ratio. It is interesting to note that the lower limit of usefulness of the hot wire is very little dependent on diameter. This discussion applies, of course, only to wires of large aspect ratio, where the heat flow is two-dimensional. For sufficiently small aspect-ratio wires three-dimensional heat

† It has recently been pointed out by Dr J. J. Mahony that this conclusion could be derived by the method of Appendix B if the heat transfer relations used are the Oseen solution (10) and Mahony's asymptotic formula for free convection at very low Grashof numbers (Mahony 1956).

transfer may modify the heat flux at Reynolds numbers higher than those defined by equation (18).

Since the preceding discussion is based on that segment of the experimental results for which errors were greatest, the quantitative aspects should be regarded as being essentially exploratory in nature, and the results treated with appropriate reserve.

4.5. *Effect of orientation*

The orientation with respect to the vertical, of the hot wire and of the plane of the airstream, is important in free convection and consequently has a bearing on the interaction of free and forced convection. As already stated, the preceding discussion (§§ 4.1 to 4.4) refers only to horizontal wires in a horizontal stream. It has been shown (Collis & Williams 1954) that in free convection two-dimensional heat transfer from vertical wires at very low Grashof numbers ($< 10^{-4}$) is not attained even for aspect ratios as large as 20,000. It is certain therefore that the aspect ratio would be significant in the case of mixed free and forced convection from vertical wires, for all practical values of the aspect ratio. The limited accuracy of the equipment available for very low Reynolds number observations did not justify a detailed study of this complicated phenomenon. However, it has been demonstrated previously (Ower 1949; Simmons 1949) that the region of interaction between buoyancy effects and forced convection extends over a much smaller velocity range for vertical wires than it does for horizontal wires. It may therefore be predicted that the forced convection heat-transfer coefficients for vertical and horizontal wires will not differ sensibly for all Reynolds numbers substantially greater than the value defined by equation (18). Some measurements which bear out this conclusion have been published by Simmons (1949), and further evidence is presented in figure 10 of this paper. Heat transfer measurements from a single wire of large aspect ratio ($l/d = 5400$) were made for both horizontal and vertical orientations. No significant difference was found within the Reynolds number range (about 0·25 to 4) which was covered. A systematic discrepancy between the two orientations, rising to $1\frac{1}{2}$ % at the highest Reynolds number is discernible, but is within the limit of reproducibility imposed by changes in the wire due to strain and annealing. The effect of orienting the airstream in other than a horizontal plane has not been examined at all. It is clear that with a vertical stream the mechanism of the interaction between free and forced convection will differ from that occurring in a horizontal stream, since buoyancy forces will be acting in a line parallel to the free stream instead of normal to it. The ambiguity in Nusselt number is unlikely to be present in the case of an upward directed stream, but will undoubtedly exist for a stream directed downwards.

4.6. *Effect of turbulence*

In the low-turbulence airstream of the wind-tunnel the onset of periodic vortex shedding was just detectable at $R = 46$. A small increase in Reynolds number (about 5 %) caused the intensity of the velocity fluctuation to rise sharply to a level which remained roughly constant as the Reynolds number was further increased. The kink in the heat transfer curve was estimated to occur at $R = 44$, which agrees well with the start of eddy shedding.

Square-mesh turbulence grids of mesh size $\frac{1}{8}$, $\frac{1}{2}$ and $\frac{15}{16}$ in. were placed in turn across the stream 10 in. ahead of the 0·002 in. (0·00535 cm) wire. The turbulence level was thus increased in steps (Batchelor & Townsend 1948) whilst maintaining the scale of small eddies (the microscale λ) about the same. Observations of the vortex street became steadily more difficult but vortex shedding still

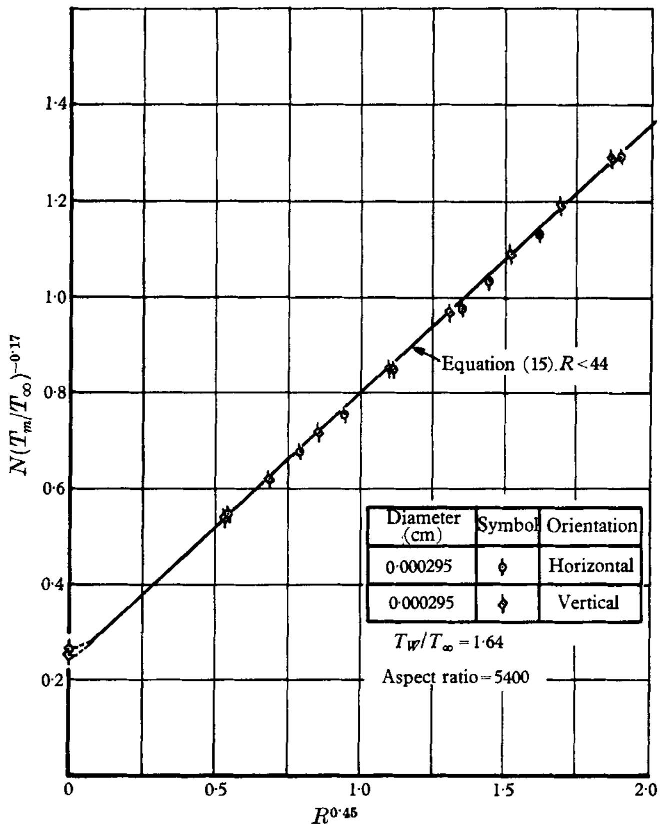


FIGURE 10. Effect of orientation on heat transfer.

commenced close to $R = 46$. The frequency of vortex shedding became increasingly uncertain, due apparently to frequency modulation arising from the fluctuating stream velocity, and to lateral displacements of the wake as a whole. However, no significant change in the Strouhal number was observed. Since the maximum effect of turbulence on the detachment of the vortex layer from the cylinder might be expected to occur with high intensity and small scale turbulence, the $\frac{1}{8}$ in. mesh was installed a distance of 20 mesh lengths upstream. Again no significant effect was observed.

It is perhaps of some interest to note that the vortex shedding frequency in these observations varied from 27 to 68 kilocycles per second.

5. Further discussion and comparison with earlier work

The experiments were designed to provide information pertaining to two-dimensional heat transfer. The results show that the effect of aspect ratio is certainly small, since agreement within the limits of error is obtained between wires of aspect ratio varying from 2070 to 8660. Again, the agreement with the Oseen solution for two-dimensional convection at very low Reynolds numbers (§ 4.3) is indicative that three-dimensional effects are small even at the lowest Reynolds numbers involved, although this situation cannot, of course, continue to indefinitely small Reynolds numbers.

A comparison with representative results taken from King (1914) and Hilpert (1933) is made in figure 11. The first impression is of haphazard scatter, but closer

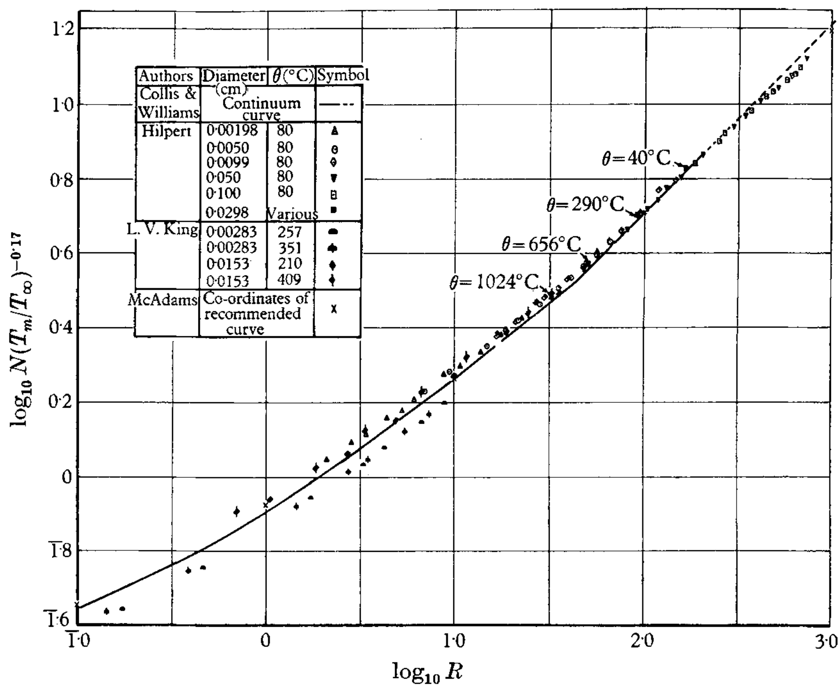


FIGURE 11. Comparison with previous forced convection investigations.

study shows that the differences between results are very largely systematic. Hilpert's work shows a high degree of self-consistency, and while exhibiting similar trends to the authors' curve the mean slope of a curve fitted to Hilpert's results would be somewhat less. Thus, while there is good agreement in values of N in the vicinity of $R = 100$, Hilpert's heat transfer is some 8 % greater at $R = 2$, and at high values of R Hilpert's curve falls below the authors' extrapolated curve. The change of slope of the $N-R$ relationship due to establishment of a vortex street is discernible at about the same point ($R \doteqdot 40$) as the bend in the authors' curve. The latter does not exhibit a gradual bend at $R = 4$ as found by Hilpert, but the occurrence of this bend was deduced from an insubstantial number of data. Where the vortex street exists, the plot of Hilpert's results shows an irregularity involving at least one point of inflexion in the range $40 < R < 500$ which is not reflected in his constants of table 1. Unfortunately, the authors' results are too sparse in this range to clarify the matter. The correct relation for this range therefore remains in some doubt. Hilpert's data below the eddy-shedding Reynolds numbers conform well to a law involving the 0·45 power of the Reynolds number as shown in figure 12. However, the constants differ from those given in table 3, for equation (15), particularly the constant, A.

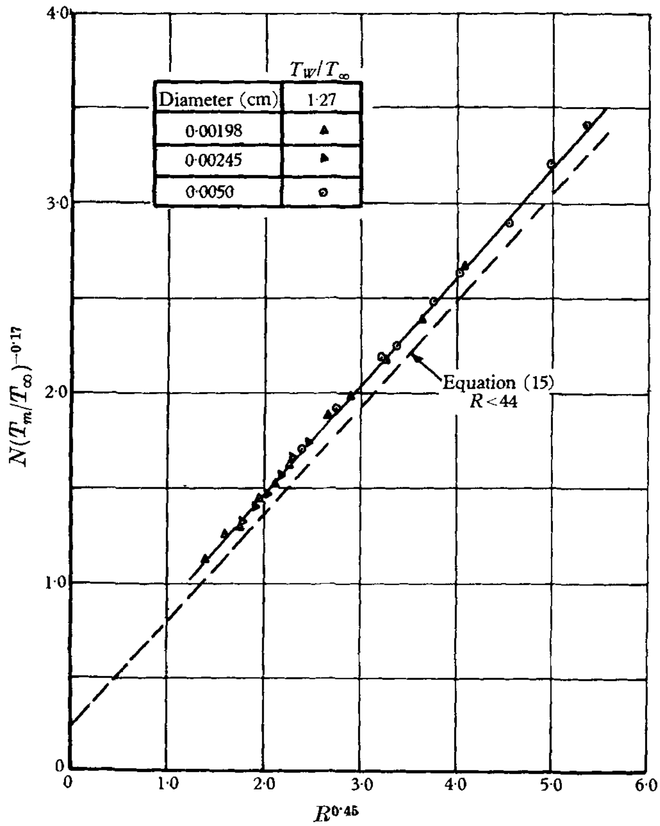


FIGURE 12. Comparison of Hilpert's data with equation (15); $R < 44$.

The data taken from King and plotted in figure 11 represent results for the lightest and heaviest wire gauges used by that author, two moderate temperature loadings being selected in each case. Near $R = 1$, heat transfer from the thicker wire is undoubtedly augmented slightly by the effect of buoyancy (see § 4.4). Apart from this it is remarkable that each series of points (of given diameter and temperature loading) is spaced a constant distance in the ordinate direction from each other and from the authors' curve. These discrepancies are evidently due to some deficiency in the measurement of the heat-transfer coefficient rather than of the Reynolds number. Regardless of the precise nature of this deficiency, it may be concluded that King's heat-transfer coefficients show the same form of dependence on the Reynolds number in the range $0·1 < R < 40$ as the work described here.

With regard to the effect of temperature loading, the authors' results constitute the only extensive body of consistent data available. The development of a method of correlating the heat-transfer coefficient with temperature loading was described in § 4.2 (equations (13), (14)). The method satisfactorily accounts for the few measurements taken by Hilpert at temperatures other than 100 °C. This can be seen from figure 11 where representative points at temperature loadings up to 1024 °C are included. The effectiveness of equations (13) and (14) in dealing with King's results cannot be determined due to the uncertainties in those results. King's own analysis showed deviations of the temperature coefficients γ and δ of equation (6) of up to 50 % from their mean value. For $R < 44$, equation (15) can be reduced to a dimensional form equivalent to King's equation (6), viz.

$$H = A'[1+\gamma(T_W - T_\infty)] + B'[1+\delta(T_W - T_\infty)]\,(Vd)^{0\cdot45}, \tag{20}$$

where A' and B' embody physical properties of the fluid and $T_\infty \doteqdot 290\,^\circ\mathrm{K}$. No significance attaches to differences between the values of A' and B' and the constants A and B of King's equation because of the changed power of Vd in the second term, but it is of interest to compare the representative values of γ and δ. The authors' values given in table 4 together with those of King are calculated assuming the following variation of conductivity and viscosity with temperature:

$$\frac{k}{k_0} = \left(\frac{T}{T_0}\right)^{0\cdot80}; \quad \frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^{0\cdot76} \tag{21}$$

The larger values found here have some significance in practical hot wire anemometry, as discussed by Collis (1956).

	γ	δ
King	0·00114	0·00008
Collis & Williams	0·00164	0·00025

TABLE 4

In the lower end of the Reynolds number range (R of order 1), comparison must be made with the work of Cole & Roshko (1954). Reference has already been made to the fact that good agreement has been found with the Oseen solution (equation (10)) obtained by these authors. Cole & Roshko did not find such agreement, but their experimental accuracy may have suffered greatly through use of nominal values of wire diameter and of temperature coefficient of resistance. Again, although very fine wires involving Knudsen numbers of up to $\frac{1}{8}$ were used, no account was taken of 'rarefaction' effects. According to equation (12) these might exceed 30 % of the measured heat transfer coefficient. In view of this, a direct comparison with the authors' results is not possible.

Some interesting indirect evidence consistent with the 0·45 power of R or V in equations (15) and (20), respectively, is found in measurements by Newman (1951) of the sensitivity of hot wire anemometers to yaw. It was originally accepted that a hot wire responded to the normal component of velocity $V \sin\psi$, where ψ is the angle of yaw, so that according to equation (6) the heat-transfer coefficient varied linearly with $\sqrt{(V \sin \psi)}$. Newman reached the conclusion, however, that the wire responded to $V^{\frac{1}{2}}(\sin \psi)^{0.457}$. In the light of equation (20) and a close examination of Newman's data, it seems probable that the wire actually responds to the 0·45 power of the normal component of velocity, i.e. $(V \sin \psi)^{0.45}$.

Conclusions

1. Based on experiments in the Reynolds number range 0·01 to 140, a new relation for two-dimensional forced convection from cylinders normal to an airstream has been established. The law has the form

$$N\left(\frac{T_m}{T_\infty}\right)^{-0.17} = A + BR^n, \tag{15}$$

where the values of n, A, B depend on whether the Reynolds number is above or below the value for which a vortex street exists in the wake of the cylinder (see table 3). Fluid properties k, ρ and μ used in computing N and R are evaluated at mean film temperature T_m, and T_∞ is in the normal range of room temperatures.

2. The extensively used relation of King was based on measurements made at Reynolds numbers below this critical value. In that region the new relation differs from King's in that the Reynolds number enters to the 0·45 power rather than the 0·5 power, and the effect of temperature loading is found to be significantly larger. These differences, particularly the former, appear to arise mainly in the analysis of the results rather than in the measurements themselves.

3. Hilpert's measurements at low Reynolds numbers ($R < 44$) are satisfied by equation (15), except for the value of the constant A. Hilpert's temperature function differs in form from that in equation (15), but no data available are sufficiently accurate to discriminate between them. For $R > 44$, there is a discrepancy between Hilpert's results and those of the authors which cannot be resolved without further investigation.

4. The theoretical heat transfer relation based on the Oseen approximation (equation (10)) is approached asymptotically as $R \to 0$, provided free-convection or aspect-ratio effects do not intervene. A relation of similar form (equation (16)) satisfactorily describes two-dimensional forced convection for $R < 0.5$.

5. Free convection effects die out quickly with increasing Reynolds number (at least for horizontal airstreams), so that except at very low velocities, the orientation of the wire with respect to the vertical has negligible influence on the heat transfer coefficient.

6. A rough criterion for the onset of buoyancy effects has been derived for horizontal wires (equation (18)). This gives quantitative expression to earlier observations that the minimum speed $V_{\min}$ which can be measured with a hot-wire anemometer increases with temperature loading, and also shows that $V_{\min}$ varies only as the $\frac{1}{5}$-power of the wire diameter.

7. Molecular effects reduce the heat transfer from fine wires below the continuum value (equation (15)) by an amount which can be estimated by assuming that the temperature differential is reduced by the 'temperature jump' calculated from kinetic theory.

8. The intensity and scale of the turbulence in the airstream does not affect the Reynolds number at which the vortex street develops in the wake of fine wires. Thus the change in constants of equation (15) likewise occurs at the same Reynolds number ($R = 44$) regardless of the stream turbulence.

The authors are indebted to the Chief Scientist, Australian Defence Scientific Service, Department of Supply, Melbourne, for permission to publish this paper.

Appendix A

Effect of temperature jump on the heat-transfer coefficient

It has been established (Kennard 1938) that when thermal conduction takes place between a rarefied gas and a bounding wall, there is a discontinuity in temperature at the wall. If the wall temperature is T_W, and T_s is what the temperature of the gas would be if the temperature gradient along the outward-drawn normal, $\partial T/\partial r$, continued right up to the wall, then the discontinuity, or temperature jump, is given by

$$T_W - T_s = -\xi \frac{\partial T}{\partial r}. \tag{A 1}$$

The constant ξ has dimensions of length and is known as the temperature jump distance.

Two assumptions are made in calculating the continuum value of the heat-transfer coefficient from the values measured in the presence of temperature jump: (*a*) the measured rate of heat transfer is the same as would take place from a cylinder of the same diameter, at a temperature T_s, immersed in a perfectly continuous gas; (*b*) the correction can be made directly to the measured value of the heat-transfer coefficient, which is of course an average of the local value taken over the circumference of the cylinder. Kennard derives an expression for ξ in terms of the properties of the gas and the surface:

$$\xi = \frac{2-\alpha}{\alpha} \frac{4c}{\gamma+1} \frac{k}{\mu c_v} L, \tag{A 2}$$

where α is the accommodation coefficient of the surface for the particular gas, k is the thermal conductivity of the gas, $\gamma = c_p/c_v$ is the ratio of specific heats of the gas at constant pressure and constant volume, μ is the viscosity of the gas, and L is the mean free path of gas molecules. Here L is defined by the relation $\mu = c\rho\bar{v}L$, in which $\bar{v} = \sqrt{(8RT/\pi)}$, where R is the gas constant per unit mass, ρ is the density of the gas, and c is a constant such that $0{\cdot}491 \leqslant c \leqslant 0{\cdot}499$. Rearranging and putting $4c = 2$, we get

$$\xi = \frac{2\gamma}{\gamma+1} \frac{1}{\sigma} \frac{2-\alpha}{\alpha} L, \tag{A 3}$$

where $\sigma = \mu c_p/k$ is the Prandtl number.

For air, $\gamma = 1{\cdot}4$ and $\sigma = 0{\cdot}72$, and for platinum in air $\alpha \doteqdot 0{\cdot}9$, so that, with little error,

$$\Delta\theta = T_W - T_s = -2L\frac{\partial T}{\partial r}. \tag{A 4}$$

If a wire is maintained at a temperature loading $\theta_W = T_W - T_\infty$, and q the heat loss per unit area is measured, then the heat-transfer coefficient h_W is calculated from the relation

$$q = h_W \theta_W, \tag{A 5}$$

and on the basis of assumptions (*a*) and (*b*) above it follows that the continuum heat-transfer coefficient h_s is given by

$$q = h_s \theta_s. \tag{A 6}$$

Combining (A 5) and (A 6), we obtain

$$h_s = h_W \left(1 + \frac{\Delta\theta}{\theta_s}\right), \tag{A 7}$$

where $\Delta\theta = T_W - T_s = \theta_W - \theta_s$.
From the conduction equation, we have

$$q = -\left(k \frac{\partial T}{\partial r}\right)_{T_s}, \tag{A 8}$$

and evaluating the quantities in equation (A 4) at temperature T_s, we also have

$$\Delta\theta = -2\left(L \frac{\partial T}{\partial r}\right)_{T_s} \tag{A 9}$$

By eliminating q and $\partial T/\partial r$ from equations (A 6) to (A 9), it can be shown that

$$h_s = h_W \left[1 + \left(\frac{2L}{k}\right)_{T_s} h_s\right], \tag{A 10a}$$

which may be written alternatively as

$$\frac{1}{h_s} = \frac{1}{h_W} - 2\left(\frac{L}{k}\right)_{T_s} \tag{A 10b}$$

In figure 13, $2L/k$ has been plotted against $T - T_0$, using values of k and L taken from Kannuluik & Carman (1951) and Kennard (1938). The correct value of $1/h_s$, must be obtained by successive approximation, starting with $T_s = T_W$.

Actually the maximum correction to the present results amounted to 11 %, giving a jump of about 30° at a wire temperature of 300 °C. In this case the error in h_s arising from evaluating $2L/k$ at T_W instead of T_s is less than $\frac{1}{2}$ %.

In order to determine the continuum Nusselt number N_C from h_s, the thermal conductivity at the mean film temperature $T_m = T_\infty + \frac{1}{2}\theta_s$ must be substituted in the expression

$$N_C = \frac{h_s d}{k_m} \tag{A 11}$$

In the example quoted above, the adjustment to k_m arising from the diminished temperature loading amounts to -2 %, so that the continuum Nusselt number (calculated from (A 11)) is 13 % greater than the measured value.

At low temperature loadings although the jump correction $\Delta\theta/\theta_s$ may be large, the absolute temperatures T_W, T_s and T_∞ are very nearly equal. If $2L/k$ is evaluated at T_∞ then

$$N_C = \frac{h_s d}{k}. \tag{A 12}$$

The measured Nusselt number

$$N_M = \frac{h_W d}{k}, \tag{A 13}$$

and substituting in (A 10*b*) we have the simple non-dimensional formula

$$\frac{1}{N_C} = \frac{1}{N_M} - 2K, \tag{A 14}$$

where $K = L/d$ = Knudsen number.

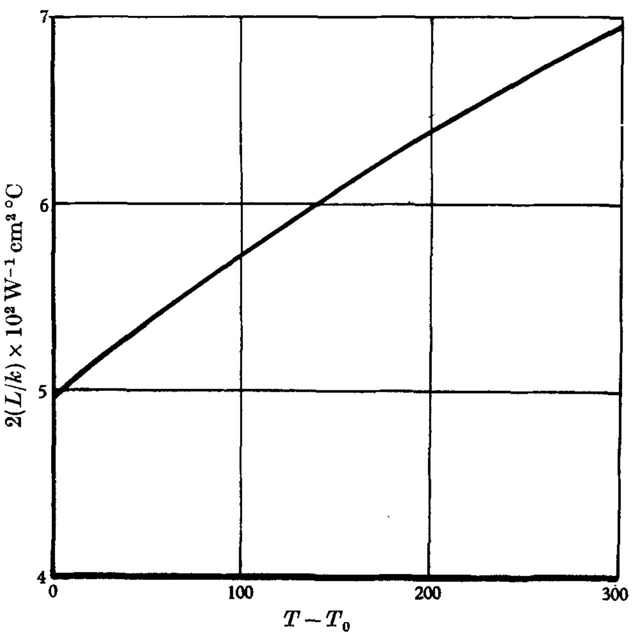


FIGURE 13. Variation of $2L/k$ with temperature for air at atmospheric pressure.

Appendix B *Criterion for the onset of buoyancy effects*

This is derived for horizontal wires on the assumption that there is no deviation from the forced convection relation for low Reynolds numbers, equation (16), until the Reynolds number is reduced to the point where the forced convection Nusselt number equals that due to free convection alone. The experimental results of this paper show that this is substantially true.

In the very low Reynolds number range where buoyancy effects are likely to be significant, the forced convection relation (see § 4.3) is

$$\frac{1}{N}\left(\frac{T_m}{T_\infty}\right)^{0 \cdot 17} = 1 \cdot 18 - 1 \cdot 10 \log_{10} R. \tag{B 1}$$

In the range $10^{-10} < G_\infty < 10^{-2}$, free convection data of Collis & Williams (1954) have been re-examined in the light of Mahony's (1956) theoretical work. It was found that a semi-logarithmic relation similar to equation (B 1) fits the experimental results very satisfactorily:

$$N^{-1} = 0 \cdot 88 - 0 \cdot 43 \log_{10} G_\infty. \tag{B 2}$$

In this relation the temperature loading has been eliminated by the method of correlation employed, previously discussed at some length by the authors (1954). Equating the Nusselt number in equations (B 1) and (B 2) yields a criterion of the type required, i.e.

$$\log R = 0{\cdot}39\left(\frac{T_m}{T_\infty}\right)^{0\cdot17}\log G_\infty + 1{\cdot}07 - 0{\cdot}80\left(\frac{T_m}{T_\infty}\right)^{0\cdot17} \tag{B 3}$$

This is not a convenient equation to deal with, but fortunately a simpler approximate result can be obtained. In the range $10^{-2} < R < 0{\cdot}2$ the temperature function of equation (B 1) may be omitted if McAdams's correlation (see § 2) is used (see note below). If R_{Mc} denotes Reynolds number evaluated in the manner of McAdams, then in lieu of equation (B 3) we obtain the condition

$$\log R_{Mc} = 0{\cdot}39 \log G_\infty + 0{\cdot}27, \tag{B 4}$$

or

$$R_{Mc} = 1{\cdot}85\, G_\infty^{0\cdot39}. \tag{B 5}$$

If the viscosity-temperature relation be taken as $\mu \propto T^{0\cdot76}$, the relation connecting R_∞ and R_{Mc} is

$$R_{Mc} = R_\infty\left(\frac{T_m}{T_\infty}\right)^{-0\cdot76} \tag{B 6}$$

Equation (B 5) can therefore be written as

$$R_\infty = 1{\cdot}85\, G_\infty^{0\cdot39}\left(\frac{T_m}{T_\infty}\right)^{0\cdot76}, \tag{B 7}$$

whence

$$V_{\min} \doteqdot \text{const.} \times d^{\frac{1}{5}}(T_W - T_\infty)^{\frac{2}{5}}\left(\frac{T_m}{T_\infty}\right)^{\frac{4}{5}}, \tag{B 8}$$

where $V_{\min}$ is the lowest velocity that can be measured without ambiguity by a hot-wire anemometer and the constant depends on the flow conditions (ρ, μ, T_∞).

Note on McAdams correlation

A simple power law can be fitted locally to any part of the $\log N$ *vs* $\log R$ curve by determining the local slope p. Thus

$$N\left(\frac{T_m}{T_\infty}\right)^{-0\cdot17} = \text{const.} \times R^p. \tag{B 9}$$

A relation connecting R and R_{Mc} may be written as

$$R = R_{Mc}\left(\frac{T_m}{T_\infty}\right)^{-1\cdot0}, \tag{B 10}$$

and equation (B 9) may be written

$$N = \text{const.} \times R_{Mc}^p\left(\frac{T_m}{T_\infty}\right)^{0\cdot17-p} \tag{B 11}$$

It can be seen that, provided p lies near $0{\cdot}17$, the temperature function is nearly annulled.

The slope p may be determined by graphical methods or by differentiation of the empirical law which describes the particular range of interest. Upon differentiating (B 1) and obtaining p, it is found that in the range $10^{-2} < R < 10^{-1}$ McAdams system will correlate data to within 1 % at temperatures up to 300 °C.

REFERENCES

BATCHELOR, G. K. & TOWNSEND, A. A. 1948 Decay of isotropic turbulence in the initial period. *Proc. Roy. Soc.* A, **193**, 539–58.

BECKERS, H. L., TER HAAR, L. W., TJOAN, LIE TIAM, MERK, H. J., PRINS, J. A. & SCHENK, J. 1956 Heat transfer at very low Grashof and Reynolds numbers. *Appl. Sci. Res.* A, **6**, 82.

COLE, J. & ROSHKO, A. 1954 Heat transfer from wires at Reynolds numbers in the Oseen range. *Proc. Heat Transfer and Fluid Mechanics Inst. Univ. of Calif. Berkeley, Calif.*

COLLIS, D. C. 1956 Forced convection of heat from cylinders at low Reynolds numbers. *J. Aero. Sci.* **23**, 697–8 (Readers' Forum).

COLLIS, D. C. & WILLIAMS, M. J. 1954 Free convection of heat from fine wires. *A.R.L. Aero.* note 140.

CORRSIN, S. 1949 Extended applications of the hot-wire anemometer. *N.A.C.A., T.N.* 1864.

GOLDSTEIN, S. (Editor). 1938 *Modern Developments in Fluid Dynamics.* Vols. I and II, Chaps. I and XIV. Oxford.

HILPERT, R. 1933 Wärmeabgage von geheizten Drähten und Rohren im Luftstrom. *ForschArb. IngWes.* **4**, 215–24.

KANNULUIK, W. G. & CARMAN, E. H. 1951 The temperature dependence of the thermal conductivity of air. *Aust. J. Sci. Res.* **4**, 305–14.

KAYE, G. W. C. & LABY, T. H. 1948 *Tables of Physical and Chemical Constants,* tenth edition. Longmans, Green and Co.

KENNARD, E. H. 1938 *Kinetic Theory of Gases.* New York and London: McGraw-Hill Book Co. Inc.

KING, L. V. 1914 On the convection of heat from small cylinders in a stream of fluid. Determination of convection constants of small platinum wires with application to hot-wire anemometry. *Phil. Trans.* A, **214**, 373–432.

MAHONY, J. J. 1956 Heat transfer at small Grashof numbers. *Proc. Roy. Soc.* A, **238**, 412–23.

MCADAMS, W. H. 1954 *Heat Transmission,* third edition, Chap. X. McGraw-Hill Book Co. Inc.

NEWMAN, B. G. 1951 Some contributions to the study of the turbulent boundary-layer near separation. *Australian Council for Aeronautics. Report* ACA–53.

OWER, E. & JOHANSEN, F. C. 1931 On a determination of the pitot static tube factor at low Reynolds numbers with special reference to the measurement of low air speeds. *R & M* 1437, *British A.R.C.*, Appendix IV.

OWER, E. 1949 *The Measurement of Airflow,* third edition, Chap. X. Chapman and Hall Ltd.

SIMMONS, L. F. G. & BEAVAN, J. A. 1934 Hot wire type of instrument for recording gusts. *R & M* 1615, *British A.R.C.*

SIMMONS, L. F. G. 1949 A shielded hot wire anemometer for low speeds. *J. Sci. Instrum.* **26**, 407–11.